868TUTORS

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Preparation for

High School Mathematics

Matrices

Solutions

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Math

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Instructions and Tips:

- ✓ **You have 120 minutes to complete this worksheet**
- ✓ **This worksheet consists of 15 questions**
- ✓ **Write answers in the spaces provided**
- ✓ **All working must be clearly shown**

Question 1

State the order of each matrix below:

(a) $\begin{pmatrix} 4 & 2 & 3 \\ 3 & 6 & 6 \\ 1 & 6 & 3 \end{pmatrix}$ $\textit{number of rows} \times \textit{number of columns}$

3 × 3

(b) $\begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ $\textit{number of rows} \times \textit{number of columns}$

3 × 1

(c) $\begin{pmatrix} 2 & 3 & 4 \end{pmatrix}$ $\textit{number of rows} \times \textit{number of columns}$

1 × 3

(d) $\begin{pmatrix} 3 \\ 2 \\ 3 \\ 6 \end{pmatrix}$ $\textit{number of rows} \times \textit{number of columns}$

4 × 1

(4 marks)

Question 2

State the order of the following matrices:

(a) $\begin{pmatrix} 2 & 5 \\ 2 & 6 \\ 3 & 9 \\ 1 & 2 \\ 3 & 2 \end{pmatrix}$ $\quad$ *number of rows × number of columns*

5 × 2

(b) $\begin{pmatrix} 4 & 5 \\ 4 & 5 \end{pmatrix}$ $\quad$ *number of rows × number of columns*

2 × 2

(c) $\begin{pmatrix} 2 & 3 & 4 & 6 \end{pmatrix}$ $\quad$ *number of rows × number of columns*

1 × 4

(d) $\begin{pmatrix} 3 \\ 2 \\ 3 \\ 6 \\ 7 \end{pmatrix}$ $\quad$ *number of rows × number of columns*

5 × 1

(4 marks)

Question 3

Consider the following matrices:

A = $\begin{pmatrix} 4 & 5 \\ 4 & 5 \end{pmatrix}$ **B =** $\begin{pmatrix} 1 & 3 \\ 4 & 9 \end{pmatrix}$ **C =** $\begin{pmatrix} 6 & 2 \\ 1 & 3 \end{pmatrix}$

Determine the following:

(a) A + B =

$$\begin{pmatrix} 4 & 5 \\ 4 & 5 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 4 & 9 \end{pmatrix} = \begin{pmatrix} 4+1 & 5+3 \\ 4+4 & 5+9 \end{pmatrix} = \boxed{\begin{pmatrix} \mathbf{5} & \mathbf{8} \\ \mathbf{8} & \mathbf{14} \end{pmatrix}}$$

(b) A + 2B =

$$\mathbf{A} = \begin{pmatrix} 4 & 5 \\ 4 & 5 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 1 & 3 \\ 4 & 9 \end{pmatrix}$$

$$\mathbf{2B} = \begin{pmatrix} 2\times 1 & 2\times 3 \\ 2\times 4 & 2\times 9 \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 8 & 18 \end{pmatrix}$$

$$\mathbf{A + 2B} = \begin{pmatrix} 4+2 & 5+6 \\ 4+8 & 5+18 \end{pmatrix} = \boxed{\begin{pmatrix} \mathbf{6} & \mathbf{11} \\ \mathbf{12} & \mathbf{23} \end{pmatrix}}$$

(c) 3B + A =

B = $\begin{pmatrix} 1 & 3 \\ 4 & 9 \end{pmatrix}$

3B = $\begin{pmatrix} 3 \times 1 & 3 \times 3 \\ 3 \times 4 & 3 \times 9 \end{pmatrix}$ **=** $\begin{pmatrix} 3 & 9 \\ 12 & 27 \end{pmatrix}$

A = $\begin{pmatrix} 4 & 5 \\ 4 & 5 \end{pmatrix}$

3B + A = $\begin{pmatrix} 3+4 & 9+5 \\ 12+4 & 27+5 \end{pmatrix}$ **=** $\boxed{\begin{pmatrix} \mathbf{7} & \mathbf{14} \\ \mathbf{16} & \mathbf{32} \end{pmatrix}}$

(d) 2A + 2B =

A = $\begin{pmatrix} 4 & 5 \\ 4 & 5 \end{pmatrix}$ **2A =** $\begin{pmatrix} 2 \times 4 & 2 \times 5 \\ 2 \times 4 & 2 \times 5 \end{pmatrix} = \begin{pmatrix} 8 & 10 \\ 8 & 10 \end{pmatrix}$

B = $\begin{pmatrix} 1 & 3 \\ 4 & 9 \end{pmatrix}$ **2B**= $\begin{pmatrix} 2 \times 1 & 2 \times 3 \\ 2 \times 4 & 2 \times 9 \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 8 & 18 \end{pmatrix}$

2A + 2B = $\begin{pmatrix} 8+2 & 10+6 \\ 8+8 & 10+18 \end{pmatrix} = \boxed{\begin{pmatrix} \mathbf{10} & \mathbf{16} \\ \mathbf{16} & \mathbf{28} \end{pmatrix}}$

(e) 2B + A - C =

2B = $\begin{pmatrix} 2 & 6 \\ 8 & 18 \end{pmatrix}$ **A =** $\begin{pmatrix} 4 & 5 \\ 4 & 5 \end{pmatrix}$ **C =** $\begin{pmatrix} 6 & 2 \\ 1 & 3 \end{pmatrix}$

2B + A - C = $\begin{pmatrix} 2 & 6 \\ 8 & 18 \end{pmatrix} + \begin{pmatrix} 4 & 5 \\ 4 & 5 \end{pmatrix} - \begin{pmatrix} 6 & 2 \\ 1 & 3 \end{pmatrix}$

2B + A - C = $\begin{pmatrix} 2+4-6 & 6+5-2 \\ 8+4-1 & 18+5-3 \end{pmatrix}$

2B + A − C = $\boxed{\begin{pmatrix} \mathbf{0} & \mathbf{9} \\ \mathbf{11} & \mathbf{20} \end{pmatrix}}$

(10 marks)

Question 4

Determine the values of P, Q, R and S.

(a) $\begin{pmatrix} 5 & P \\ 20 & 5 \end{pmatrix} + \begin{pmatrix} 3 & -5 \\ R & 6 \end{pmatrix} = \begin{pmatrix} Q & 10 \\ 18 & S \end{pmatrix}$

P + -5 = 10

P = 10 + 5 = 15

P =15

20 + R = 18

R = 18 - 20

R = -2

5 + 3 = Q

Q = 8

5 + 6 = S

S = 11

Determine the values of A, B, C and D.

(b) $\begin{pmatrix} 3 & A \\ 4 & 9 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} B & 3 \\ C & D \end{pmatrix}$

3 + 2 = B

B = 5

4 + 3 = C

C = 7

A + 1 = 3

A = 3 - 1

A = 2

9 + 5 = D

D = 14

(4 marks)

Question 5

Consider the following matrices:

$$\mathbf{P} = \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 9 & 10 \\ 8 & 7 \end{pmatrix} \quad \mathbf{R} = \begin{pmatrix} 11 & 12 \\ 1 & 2 \end{pmatrix}$$

(a) $Q + R = \begin{pmatrix} 9 & 10 \\ 8 & 7 \end{pmatrix} + \begin{pmatrix} 11 & 12 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 9+11 & 10+12 \\ 8+1 & 7+2 \end{pmatrix}$

$$\mathbf{Q + R} = \begin{pmatrix} \mathbf{20} & \mathbf{22} \\ \mathbf{9} & \mathbf{9} \end{pmatrix}$$

(b) P + 2Q + R =

$P = \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix}$ $2Q = \begin{pmatrix} 18 & 20 \\ 16 & 14 \end{pmatrix}$ $R = \begin{pmatrix} 11 & 12 \\ 1 & 2 \end{pmatrix}$

$P + 2Q + R = \begin{pmatrix} 1+18+11 & 2+20+12 \\ 2+16+1 & 6+14+2 \end{pmatrix} = \begin{pmatrix} 30 & 34 \\ 19 & 22 \end{pmatrix}$

$$\mathbf{P + 2Q + R} = \begin{pmatrix} \mathbf{30} & \mathbf{34} \\ \mathbf{19} & \mathbf{22} \end{pmatrix}$$

(c) R - P =

R = $\begin{pmatrix} 11 & 12 \\ 1 & 2 \end{pmatrix}$ **P =** $\begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix}$

R - P = $\begin{pmatrix} 11-1 & 12-2 \\ 1-2 & 2-6 \end{pmatrix}$

R - P = $\begin{pmatrix} \mathbf{10} & \mathbf{10} \\ \mathbf{-1} & \mathbf{-4} \end{pmatrix}$

(d) 2Q - R =

2Q = $\begin{pmatrix} 2\times 9 & 2\times 10 \\ 2\times 8 & 2\times 7 \end{pmatrix}$ = $\begin{pmatrix} 18 & 20 \\ 16 & 14 \end{pmatrix}$

$2Q = \begin{pmatrix} 18 & 20 \\ 16 & 14 \end{pmatrix}$

$2Q = \begin{pmatrix} 18 & 20 \\ 16 & 14 \end{pmatrix}$ $R = \begin{pmatrix} 11 & 12 \\ 1 & 2 \end{pmatrix}$

$2Q$ - $R = \begin{pmatrix} 18-11 & 20-12 \\ 16-1 & 14-2 \end{pmatrix}$

2Q - R = $\begin{pmatrix} \mathbf{7} & \mathbf{8} \\ \mathbf{15} & \mathbf{12} \end{pmatrix}$

(e) P + Q + R =

P = $\begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix}$ $Q = \begin{pmatrix} 9 & 10 \\ 8 & 7 \end{pmatrix}$ $R = \begin{pmatrix} 11 & 12 \\ 1 & 2 \end{pmatrix}$

P + Q + R = $\begin{pmatrix} 1+9+11 & 2+10+12 \\ 2+8+1 & 6+7+2 \end{pmatrix}$

P + Q + R = $\begin{pmatrix} \mathbf{21} & \mathbf{24} \\ \mathbf{11} & \mathbf{15} \end{pmatrix}$

(10 marks)

Question 6

Consider the following matrices:

A = $\begin{pmatrix} 1 & 5 \\ 2 & 6 \end{pmatrix}$ **B** = $\begin{pmatrix} 9 & 1 \\ 8 & 1 \end{pmatrix}$ **C** = $\begin{pmatrix} 11 & 12 \\ 1 & 1 \end{pmatrix}$

(a) Find $|\mathrm{A}|, |\mathrm{B}|, |\mathrm{C}|$

A = $\begin{pmatrix} 1 & 5 \\ 2 & 6 \end{pmatrix}$ *let* **A** = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $|\mathbf{A}|$ = *(a) (d) − (b) (c)*

$|\mathbf{A}|$ = *(1) (6) − (5) (2)* $|\mathbf{A}|$ = *6 -10*

$|\mathbf{A}|$ = -4

B = $\begin{pmatrix} 9 & 1 \\ 8 & 1 \end{pmatrix}$ *let* **B** = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $|\mathbf{B}|$ = *(a) (d) − (b) (c)*

$|\mathbf{B}|$ = *(9) (1) − (1) (8)*

$|\mathbf{B}|$ = 1

C = $\begin{pmatrix} 11 & 12 \\ 1 & 1 \end{pmatrix}$ *let* **C** = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$|\mathbf{C}|$ = *(11) (1) − (12) (1)*

$|\mathbf{C}|$ = *11-12*

$|\mathbf{C}|$= -1

(6 marks)

(b) Find A^{-1}

$\mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{(a)(d)-(b)(c)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$\mathbf{A}^{-1} = -\frac{1}{4}\begin{pmatrix} 6 & -5 \\ -2 & 1 \end{pmatrix}$ $\mathbf{A}^{-1} = \begin{pmatrix} -1.5 & 1.25 \\ 0.5 & -0.25 \end{pmatrix}$

(3 marks)

(c) Find B^{-1}

$B = \begin{pmatrix} 9 & 1 \\ 8 & 1 \end{pmatrix}$ *let* $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$B^{-1} = \frac{1}{|B|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{(a)(d)-(b)(c)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$B^{-1} = -\frac{1}{1}\begin{pmatrix} 1 & -1 \\ -8 & 9 \end{pmatrix}$

$\mathbf{B^{-1} = \begin{pmatrix} 1 & -1 \\ -8 & 9 \end{pmatrix}}$

(3 marks)

(d) Find C^{-1}

$C = \begin{pmatrix} 11 & 12 \\ 1 & 1 \end{pmatrix}$ *let* $C = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$C^{-1} = \frac{1}{|C|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{-1}\begin{pmatrix} 1 & -12 \\ -1 & 11 \end{pmatrix}$

$\mathbf{C^{-1} = \begin{pmatrix} -1 & 12 \\ 1 & -11 \end{pmatrix}}$

(3 marks)

Question 7

Multiply the following matrices. If the matrices cannot be multiplied state a reason why.

(a) $\begin{pmatrix}1 & 8\\ 2 & 6\end{pmatrix} \times \begin{pmatrix}3 & 3 & 3\\ 4 & 4 & 4\end{pmatrix} = \begin{pmatrix}1\times 3 + 8\times 4 & 1\times 3 + 8\times 4 & 1\times 3 + 8\times 4\\ 2\times 3 + 6\times 4 & 2\times 3 + 6\times 4 & 2\times 3 + 6\times 4\end{pmatrix}$

2×2 *2×3*

$\begin{pmatrix}1\times 3 + 8\times 4 & 1\times 3 + 8\times 4 & 1\times 3 + 8\times 4\\ 2\times 3 + 6\times 4 & 2\times 3 + 6\times 4 & 2\times 3 + 6\times 4\end{pmatrix} = \boxed{\begin{pmatrix}\mathbf{35} & \mathbf{35} & \mathbf{35}\\ \mathbf{30} & \mathbf{30} & \mathbf{30}\end{pmatrix}}$

(2 marks)

(b) $\begin{pmatrix}9 & 10\\ 8 & 9\end{pmatrix} \times \begin{pmatrix}11 & 12\\ 1 & 1\end{pmatrix} = \begin{pmatrix}9\times 11 + 10\times 1 & 9\times 12 + 10\times 1\\ 8\times 11 + 9\times 1 & 8\times 12 + 9\times 1\end{pmatrix}$

2×2 *2×2* $= \begin{pmatrix}99 + 10 & 108 + 10\\ 88 + 9 & 96 + 9\end{pmatrix} = \boxed{\begin{pmatrix}\mathbf{109} & \mathbf{118}\\ \mathbf{97} & \mathbf{105}\end{pmatrix}}$

(2 marks)

c) $\begin{pmatrix}3 & 3 & 3\\ 4 & 4 & 4\end{pmatrix} \times \begin{pmatrix}1 & 8\\ 2 & 6\end{pmatrix} =$

2×3 *2×2*

These matrices cannot be multiplied since the number of columns in the first matrix is not equal to number of rows in the second matrix.

(2 marks)

Question 8

P = $\begin{pmatrix} 1 & 3 \\ 1 & 6 \end{pmatrix}$ **Q =** $\begin{pmatrix} 9 & 16 \\ 1 & 2 \end{pmatrix}$ **R =** $\begin{pmatrix} 11 & 11 \\ 0 & 1 \end{pmatrix}$

(a)Find PQ

PQ = $\begin{pmatrix} 1 & 3 \\ 1 & 6 \end{pmatrix}$ **×** $\begin{pmatrix} 9 & 16 \\ 1 & 2 \end{pmatrix}$ **=** $\begin{pmatrix} 1 \times 9 + 3 \times 1 & 1 \times 16 + 3 \times 2 \\ 1 \times 9 + 6 \times 1 & 1 \times 16 + 6 \times 2 \end{pmatrix}$

PQ = $\begin{pmatrix} 9 + 3 & 16 + 6 \\ 9 + 6 & 16 + 12 \end{pmatrix}$ **PQ =** $\begin{pmatrix} \mathbf{12} & \mathbf{22} \\ \mathbf{15} & \mathbf{28} \end{pmatrix}$

(2 marks)

(b) Find QR

QR = $\begin{pmatrix} 9 & 16 \\ 1 & 2 \end{pmatrix} \times \begin{pmatrix} 11 & 11 \\ 0 & 1 \end{pmatrix}$ **=** $\begin{pmatrix} 9 \times 11 + 16 \times 0 & 9 \times 11 + 16 \times 1 \\ 1 \times 11 + 2 \times 0 & 1 \times 11 + 2 \times 1 \end{pmatrix}$

QR = $\begin{pmatrix} 99 + 0 & 99 + 16 \\ 11 + 0 & 11 + 2 \end{pmatrix}$ **QR =** $\begin{pmatrix} \mathbf{99} & \mathbf{115} \\ \mathbf{11} & \mathbf{13} \end{pmatrix}$

(2 marks)

(c) Find RQ

$$\mathbf{RQ} = \begin{pmatrix} 11 & 11 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} 9 & 16 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 11 \times 9 + 11 \times 1 & 11 \times 16 + 11 \times 2 \\ 0 \times 9 + 1 \times 1 & 0 \times 16 + 1 \times 2 \end{pmatrix}$$

$$\mathbf{RQ} = \begin{pmatrix} 99 + 11 & 176 + 22 \\ 0 + 1 & 0 + 2 \end{pmatrix}$$

$$\mathbf{RQ} = \begin{pmatrix} \mathbf{110} & \mathbf{198} \\ \mathbf{1} & \mathbf{2} \end{pmatrix}$$

(2 marks)

(d) Find PQR

$PQR = PQ \times R$

Recall from (a) $PQ = \begin{pmatrix} \mathbf{12} & \mathbf{22} \\ \mathbf{15} & \mathbf{28} \end{pmatrix}$

$$PQR = \begin{pmatrix} 12 & 22 \\ 15 & 28 \end{pmatrix} \times \begin{pmatrix} 11 & 11 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{PQR} = \begin{pmatrix} 12 \times 11 + 22 \times 0 & 12 \times 11 + 22 \times 1 \\ 15 \times 11 + 28 \times 0 & 15 \times 11 + 28 \times 1 \end{pmatrix}$$

$$PQR = \begin{pmatrix} 132 + 0 & 132 + 22 \\ 165 + 0 & 165 + 28 \end{pmatrix}$$

$$\mathbf{PQR} = \begin{pmatrix} \mathbf{132} & \mathbf{154} \\ \mathbf{165} & \mathbf{193} \end{pmatrix}$$

(2 marks)

Question 9

(a) Express the equations

2x - 6y = 0

3x + 2y = 44

in the form AX = B, where A, X and B are matrices.

$\begin{pmatrix} 2 & -6 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 44 \end{pmatrix}$

$\mathbf{A} = \begin{pmatrix} \mathbf{2} & \mathbf{-6} \\ \mathbf{3} & \mathbf{2} \end{pmatrix}$ $\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} \mathbf{0} \\ \mathbf{44} \end{pmatrix}$

(2 marks)

(b) Express the equations

4x - 3y = 5

5x - 2y = 8

in the form AX = B, where A,X and B are matrices.

$\begin{pmatrix} 4 & -3 \\ 5 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

$\mathbf{A} = \begin{pmatrix} \mathbf{4} & \mathbf{-3} \\ \mathbf{5} & \mathbf{-2} \end{pmatrix}$ $\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} \mathbf{5} \\ \mathbf{8} \end{pmatrix}$

(2 marks)

Question 10

Solve the simultaneous equations using matrices:

x + y = 14

2x + 3y = 33

$\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 14 \\ 33 \end{pmatrix}$

A X B

X = A^{-1} B

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{|A|} \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 14 \\ 33 \end{pmatrix}$

let **A** = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

|A| = (a) (d) − (b) (c) |A| = (1) (3)-(1) (2) |A| = 3-2 |A| =1

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{1} \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 14 \\ 33 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 14 \\ 33 \end{pmatrix}$

2×2 2×1

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 14 \\ 33 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \times 14 & + & -1 \times 33 \\ -2 \times 14 & + & 1 \times 33 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 42 - 33 \\ -28 + 33 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \end{pmatrix}$

x = 9

y = 5

(6 marks)

Question 11

Solve the simultaneous equations using matrices.

a + b = 10

3a + 2b = 28

$\begin{pmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{3} & \mathbf{2} \end{pmatrix} \begin{pmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{pmatrix} = \begin{pmatrix} \mathbf{10} \\ \mathbf{28} \end{pmatrix}$

A X B

X= A^{-1} B

$\begin{pmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{pmatrix} = \frac{1}{|A|} \begin{pmatrix} \mathbf{2} & \mathbf{-1} \\ \mathbf{-3} & \mathbf{1} \end{pmatrix} \begin{pmatrix} \mathbf{10} \\ \mathbf{28} \end{pmatrix}$ *let* **A** = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

|A| = *(a) (d) – (b) (c)* **|A| =** *(1) (2) - (1) (3)* **|A| = 2-3** **|A| = -1**

$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{-1} \begin{pmatrix} 2 & -1 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 28 \end{pmatrix}$ $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 10 \\ 28 \end{pmatrix}$

2×2 2×1

$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 10 \\ 28 \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -2 \times 10 & + & 1 \times 28 \\ 3 \times 10 & + & -1 \times 28 \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -20 + 28 \\ 30 - 28 \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$

a = 8

b = 2

(6 marks)

Question 12

Solve the simultaneous equations using matrices.

p – q = 10

2p + q = 26

$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 10 \\ 26 \end{pmatrix}$

A X B

X= A⁻¹ B

$\begin{pmatrix} p \\ q \end{pmatrix} = \frac{1}{|A|} \begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 26 \end{pmatrix}$ *let* **A** = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$|A| = (a)(d) - (b)(c)$ $|A| = (1)(1) - (-1)(2)$ $|A| = 1 + 2$ $|A| = 3$

$\begin{pmatrix} p \\ q \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 26 \end{pmatrix}$ $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 10 \\ 28 \end{pmatrix}$

2×2 2×1

$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \times 10 + \frac{1}{3} \times 26 \\ \frac{-2}{3} \times 10 + \frac{1}{3} \times 26 \end{pmatrix}$

$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \frac{10}{3} + \frac{26}{3} \\ \frac{-20}{3} + \frac{26}{3} \end{pmatrix}$

$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \frac{36}{3} \\ \frac{6}{3} \end{pmatrix}$

$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 12 \\ 2 \end{pmatrix}$

p = 12

q = 2

(6 marks)

Question 13

(a) The Matrix A is defined as:

A = $\begin{pmatrix} 5 & 2 \\ b & 4 \end{pmatrix}$

Determine the value of b **for which the matrix A does not have an inverse.**

Let A **=** $\begin{pmatrix} w & x \\ y & z \end{pmatrix}$ $|\mathrm{A}|$ **=** *(w) (z) − (x) (y)*

The matrix A does not have an inverse when $|\mathrm{A}|$ *= 0*

0 = (5) (4) − (2) (b)

20 -2b =0

-2b = -20

b = $\frac{20}{2}$ **b= 10** *(The matrix A does not have an inverse when b=10)*

(2 marks)

(b)The Matrix C is defined as:

C = $\begin{pmatrix} 6 & d \\ 2 & 4 \end{pmatrix}$

Determine the value of d **for which the matrix C does not have an inverse.**

Let C **=** $\begin{pmatrix} w & x \\ y & z \end{pmatrix}$ $|\mathrm{A}|$ **=** *(w) (z) − (x) (y)*

The matrix C does not have an inverse when $|\mathrm{C}|$ *= 0*

0= (6) (4) − (d) (2)

24 -2d = 0

-2d = -24

d = $\frac{-24}{-2}$ **d = 12** *(The matrix C does not have an inverse when d =12)*

(2 marks)

Question 14

(a) The Matrix Y is defined as:

Y = $\begin{pmatrix} -5 & 2 \\ p & 4 \end{pmatrix}$

Determine the value of p for which the matrix Y does not have an inverse.

Let Y **=** $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ **|Y| =** *(a) (d) − (b) (c)*

The matrix A does not have an inverse when |Y| = 0

0= (-5) (4) − (2) (p)

-20 -2p = 0

-2p = -20

$p = \frac{20}{2}$ **p = -10** *(The matrix A does not have an inverse when p= -10)*

(2 marks)

(b) The Matrix Z is defined as:

Z = $\begin{pmatrix} 10 & h \\ 9 & 18 \end{pmatrix}$

Determine the value of h for which the matrix Z does not have an inverse.

Let Z **=** $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ **|Z| =** *(a) (d) − (b) (c)*

The matrix A does not have an inverse when |Z| = 0

0 = (10) (18) − (h) (9)

180 -9h = 0

-9h = -180

$h = \frac{-180}{-9}$ **h = 20** *(The matrix A does not have an inverse when h = 20)*

(2 marks)

Question 15

(a) State the 2 × 2 transformation matrix which represents a reflection in the line y = x.

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(b) State the 2 × 2 transformation matrix which represents a reflection in the line y = -x.

$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

(c) State the 2 × 2 transformation matrix which represents a reflection in the x axis.

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(d) State the 2 × 2 transformation matrix which represents a reflection in the y axis.

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(e) State the 2 × 2 transformation matrix which represents a 90° clockwise rotation about the origin.

$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(10 marks)

END OF WORKSHEET