

868



TUTORS

Preparation for

High School Mathematics

Trigonometry

(Non-right angled triangles)

Solutions

Math



Instructions and Tips:

- ✓ **You have 90 minutes to complete this worksheet**
- ✓ **This worksheet consists of 4 questions**
- ✓ **Write answers in the spaces provided**
- ✓ **All working must be clearly shown**



Student Name: _____

Student ID: _____

Date: __ / __ / ____

Total Score:

Highest Score:

Tutor's Comments:

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Question 1

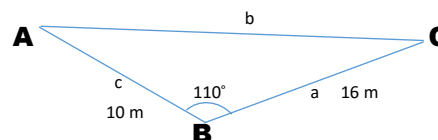
You are required to solve each triangle below. Each triangle is a non-right-angled triangle. In the space provided, sketch a triangle for reference. You need to decide whether to use the sine rule or cosine rule (or both) to solve the triangle. You can relabel the triangle where appropriate.

(a) ABC is a triangle

AB = 10 m

BC = 16 m

$\angle ABC = 110^\circ$



Solving for b (Applying cosine rule)

$$b^2 = a^2 + c^2 - 2ac \times \cos B$$

$$b^2 = (16)^2 + (10)^2 - 2(16)(10) \times \cos 110^\circ$$

$$b^2 = 256 + 100 - 320 \cos 110^\circ$$

$$b^2 = 256 + 100 - 320 \cos 110^\circ$$

$$b^2 = 356 + 320 \cos 110^\circ$$

$$b^2 = 356 + 109.4464459$$

$$b^2 = 465.4464459 \quad \boxed{b = 21.57 \text{ m (to 2 decimal places)}}$$



Solving for A (Applying cosine rule)

$$a^2 = b^2 + c^2 - 2bc \times \cos A$$

$$(16)^2 = 465.4464459 + 100 - 2(21.57420789)(10) \times \cos A$$

$$256 - 565.4464459 = -431.4841578 \cos A$$

$$-309.4464459 = -431.4841578 \cos A$$

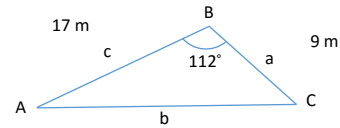
$$-431.4841578 \cos A = -309.4464459$$

$$\cos A = \frac{-309.4464459}{-431.4841578} \quad A = \cos^{-1}(0.717167572) \quad \boxed{A = 44.18^\circ \text{ (to 2 decimal places)}}$$

$$C = 180^\circ - (44.18^\circ + 110^\circ) \quad \boxed{C = 25.82^\circ \text{ (to 2 decimal places)}}$$

(5 marks)

(b) ABC is a triangle
AB = 17 m
BC = 9 m
 $\angle ABC = 112^\circ$



Solving for b (Applying cosine rule)

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$b^2 = (9)^2 + (17)^2 - 2(9)(17) \times \cos 112^\circ$$

$$b^2 = 81 + 289 - 306 \cos 112^\circ$$

$$b^2 = 370 - -114.6296176$$

$$b^2 = 370 + 114.6296176$$

$$b^2 = 484.6296176$$

$$b = \sqrt{484.6296176}$$

$$\boxed{b = 22.01 \text{ m (to 2 decimal places)}}$$



Solving for A (applying sine rule)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{9}{\sin A} = \frac{22.01}{\sin 112^\circ} \quad 22.01 \times \sin A = 9 \times \sin 112^\circ \quad \sin A = \frac{9 \times \sin 112^\circ}{22.01} \quad \sin A = \frac{8.344654691}{22.01}$$

$$\sin A = 0.379130154$$

$$A = \sin^{-1}(0.379130154)$$

$$\boxed{A = 22.28^\circ \text{ (to 2 decimal places)}}$$

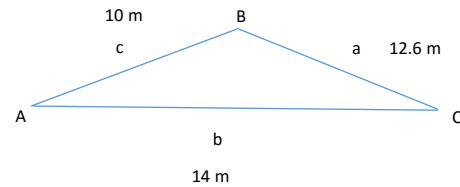
$$C = 180^\circ - (112^\circ + 22.28^\circ) \text{ (Internal angles of a triangle sum to } 180^\circ)$$

$$C = 180^\circ - 134.28^\circ$$

$$\boxed{C = 45.72^\circ \text{ (to 2 decimal places)}}$$

(5 marks)

(c) ABC is a triangle
AB = 10 m
AC = 14 m
BC = 12.6 m



Solving for A (Applying cosine rule)

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$(12.6)^2 = (14)^2 + (10)^2 - 2(14)(10) \times \cos A$$

$$(12.6)^2 = 196 + 100 - 280 \cos A$$

$$158.76 - 296 = -280 \cos A$$

$$-280 \cos A = -137.24$$

$$\cos A = \frac{-137.24}{-280}$$

$$A = \cos^{-1} \left(\frac{-137.24}{-280} \right)$$

$$A = \cos^{-1} (0.490142857)$$

$$\boxed{A = 60.65^\circ \text{ (to 2 decimal places)}}$$

Solving for C (Applying cosine rule)

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$(10)^2 = (12.6)^2 + (14)^2 - 2(12.6)(14) \times \cos C$$

$$100 = 158.76 + 196 - 352.8 \times \cos C$$

$$100 = 158.76 + 196 - 352.8 \times \cos C$$

$$100 = 354.76 - 352.8 \times \cos C$$

$$100 - 354.76 = -352.8 \times \cos C$$

$$-352.8 \times \cos C = -254.76$$

$$\cos C = \frac{-254.76}{-352.8} \quad \cos C = 0.722108843$$

$$C = \cos^{-1} (0.722108843)$$

$$\boxed{C = 43.77^\circ \text{ (to 2 decimal places)}}$$

Solving for B (Internal angles in a triangle sum to 180°)

$$B = 180^\circ - (60.65^\circ + 43.77^\circ)$$

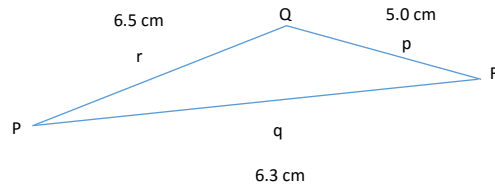
$$B = 180^\circ - (104.42^\circ)$$

$$\boxed{B = 75.58^\circ \text{ (to 2 decimal places)}}$$

(5 marks)



(d) PQR is a triangle
PR = 6.3 cm
PQ = 6.5 cm
QR = 5.0 cm



Solving for P (Applying cosine rule)

$$p^2 = q^2 + r^2 - 2qr \cos P$$

$$(5)^2 = (6.3)^2 + (6.5)^2 - 2(6.3)(6.5) \cos P$$

$$25 = 39.69 + 42.25 - 81.9 \cos P$$

$$25 = 81.94 - 81.9 \cos P$$

$$25 - 81.94 = -81.9 \cos P$$

$$-81.9 \cos P = -56.94$$

$$\cos P = \frac{-56.94}{-81.9}$$

$$\cos P = 0.695238095$$

$$P = \cos^{-1}(0.695238095)$$

$$\boxed{P = 45.95^\circ \text{ (to 2 decimal places)}}$$



Solving for R (Applying Cosine rule)

$$r^2 = p^2 + q^2 - 2pq \cos R$$

$$(6.5)^2 = (5)^2 + (6.3)^2 - 2(5)(6.3) \cos R$$

$$42.25 = 25 + 39.69 - 63 \cos R$$

$$42.25 = 64.69 - 63 \cos R$$

$$-63 \cos R = 42.25 - 64.69$$

$$-63 \cos R = -22.44$$

$$\cos R = \frac{-22.44}{-63} \quad \cos R = 0.356190476$$

$$R = \cos^{-1}(0.356190476)$$

$$\boxed{R = 69.13^\circ \text{ (to 2 decimal places)}}$$

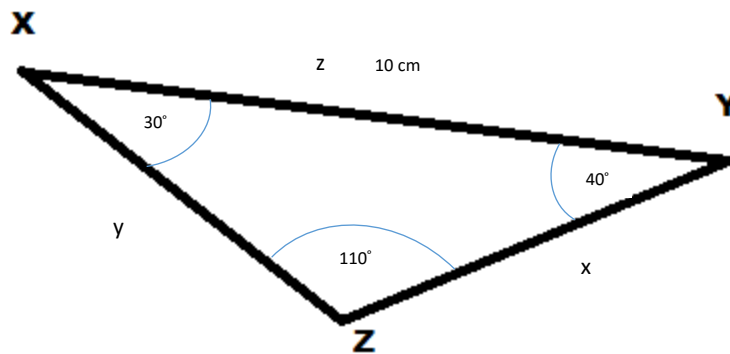
Solving for Q (Internal angles in a triangle sum to 180°)

$$Q = 180^\circ - (69.13^\circ + 45.95^\circ)$$

$$Q = 180^\circ - (115.08^\circ)$$

$$\boxed{Q = 64.92^\circ \text{ (to 2 decimal places)}}$$

(5 marks)

Question 2**Solve each triangle below. (Relabel if necessary)****(a) XY = 10 cm** **$\angle YXZ = 30^\circ$** **$\angle XYZ = 40^\circ$** Internal angles in a triangle sum to 180°

$$Z = 180^\circ - (40^\circ + 30^\circ)$$

$$Z = 180^\circ - (70^\circ) \quad Z = 110^\circ$$

Solving for x (Applying sine rule)

$$\frac{x}{\sin x} = \frac{y}{\sin y} = \frac{z}{\sin z}$$

$$\frac{x}{\sin x} = \frac{z}{\sin z} \quad \frac{x}{\sin 30^\circ} = \frac{10}{\sin 110^\circ} \quad x \sin 110^\circ = 10 \sin 30^\circ$$

$$x \sin 110^\circ = 10 \sin 30^\circ$$

$$x = \frac{10 \sin 30^\circ}{\sin 110^\circ}$$

$$x = \frac{5}{0.93969262}$$

$$\boxed{x = 5.32 \text{ cm (to 2 decimal places)}}$$

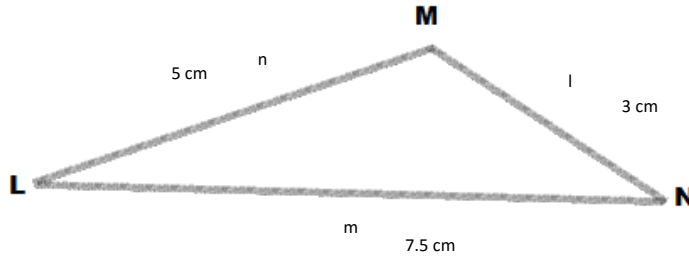
Solving for y (Applying sine rule)

$$\frac{y}{\sin y} = \frac{z}{\sin z} \quad \frac{y}{\sin 40^\circ} = \frac{10}{\sin 110^\circ}$$

$$y \sin 110^\circ = 10 \sin 40^\circ \quad y = \frac{10 \sin 40^\circ}{\sin 110^\circ} \quad y = \frac{6.427876097}{0.93969262} \quad \boxed{y = 6.84 \text{ cm (to 2 decimal places)}}$$

(5 marks)

- (b) LM = 5 cm**
LN = 7.5 cm
MN = 3 cm



Solving for L (applying cosine rule)

$$l^2 = m^2 + n^2 - 2mn \cos L$$

$$(3)^2 = (7.5)^2 + (5)^2 - 2(7.5)(5) \cos L$$

$$9 = 56.25 + 25 - 75 \cos L$$

$$9 = 81.25 - 75 \cos L$$

$$81.25 - 75 \cos L = 9$$

$$-75 \cos L = 9 - 81.25$$

$$-75 \cos L = -72.25$$

$$\cos L = \frac{-72.25}{-75} \quad \cos L = 0.963333333 \quad L = \cos^{-1}(0.963333333)$$

$$\boxed{L = 15.56^\circ \text{ (to 2 decimal places)}}$$

Solving for N (Applying cosine rule)

$$n^2 = m^2 + l^2 - 2ml \cos N$$

$$(5)^2 = (7.5)^2 + (3)^2 - 2(7.5)(3) \cos N$$

$$25 = 56.25 + 9 - 45 \cos N$$

$$25 = 65.25 - 45 \cos N$$

$$65.25 - 45 \cos N = 25$$

$$-45 \cos N = 25 - 65.25$$

$$-45 \cos N = -40.25$$

$$\cos N = \frac{-40.25}{-45} \quad \cos N = 0.894444444$$

$$N = \cos^{-1}(0.894444444)$$

$$\boxed{N = 26.56^\circ \text{ (to 2 decimal places)}}$$

Internal angles in a triangle sum to 180°

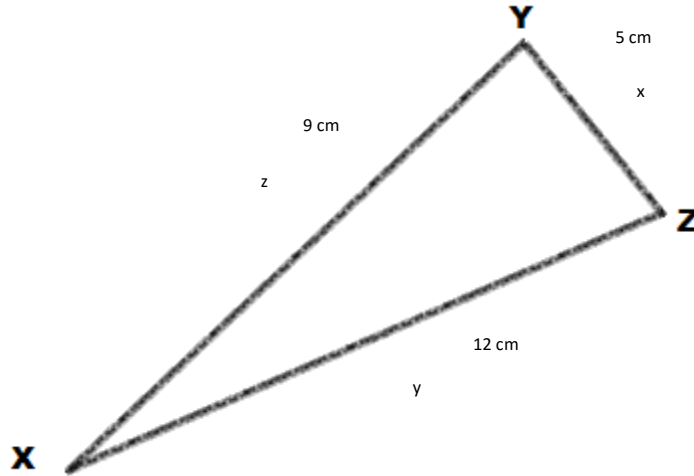
$$M + N + L = 180^\circ$$

$$M = 180^\circ - (N + L) \quad M = 180^\circ - (15.56^\circ + 26.56^\circ) = 137.88^\circ$$

$$\boxed{M = 137.88^\circ \text{ (to 2 decimal places)}}$$

(5 marks)

- (c) **XY = 9 cm**
XZ = 12 cm
YZ = 5 cm



Solving for X (Applying cosine rule)

$$x^2 = y^2 + z^2 - 2yz \cos X$$

$$(5)^2 = (12)^2 + (9)^2 - 2(12)(9) \cos X \quad (5)^2 = 144 + 81 - 2(12)(9) \cos X$$

$$(5)^2 = 144 + 81 - 216 \cos X \quad 25 = 225 - 216 \cos X$$

$$225 - 216 \cos X = 25$$

$$-216 \cos X = 25 - 225$$

$$-216 \cos X = -200$$

$$\cos X = \frac{-200}{-216} = 0.925925925 \quad X = \cos^{-1}(0.925925925) \quad \boxed{X = 22.19^\circ \text{ (to 2 decimal places)}}$$

Solving for Y (Applying sine rule)

$$\frac{y}{\sin Y} = \frac{x}{\sin X}$$

$$\frac{12}{\sin Y} = \frac{5}{\sin 22.19160657}$$

$$5 \sin Y = 12 \sin 22.19160657$$

$$\sin Y = \frac{12 \sin 22.19160657}{5} = 0.906492358$$

$$Y = \sin^{-1}(0.906492358) \quad \boxed{Y = 65.03^\circ \text{ (to 2 decimal places)}}$$

Solving for Z (Internal angles in a triangle sum to 180°)

$$Z = 180^\circ - (65.03^\circ + 22.19^\circ)$$

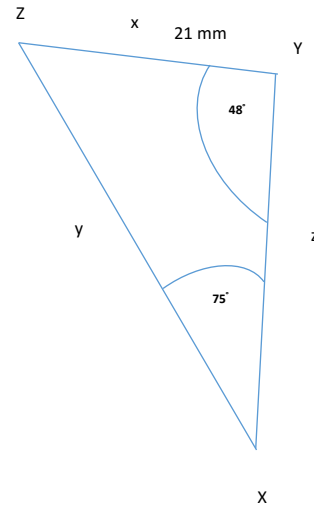
$$Z = 180^\circ - 87.22^\circ \quad \boxed{Z = 92.78^\circ \text{ (to 2 decimal places)}}$$

(5 marks)

Question 3

Solve each triangle. Sketches should be drawn for reference. Relabel where necessary.

- (a) Triangle XYZ**
 $X = 75^\circ$ $Y = 48^\circ$ $x = 21\text{mm}$



Internal angles in a triangle sum to 180°

$$Z = 180^\circ - (75^\circ + 48^\circ)$$

$$\boxed{Z = 57^\circ}$$



Solving for y (Applying sine rule)

$$\frac{y}{\sin Y} = \frac{x}{\sin X} \quad \frac{y}{\sin 48^\circ} = \frac{21}{\sin 75^\circ}$$

$$y \sin 75^\circ = 21 \sin 48^\circ$$

$$y = \frac{21 \sin 48^\circ}{\sin 75^\circ} \quad y = \frac{15.60604134}{0.965925826}$$

$$\boxed{y = 16.16 \text{ mm (to 2 decimal places)}}$$

Solving for z (Applying sine rule)

$$\frac{z}{\sin Z} = \frac{x}{\sin X} \quad \frac{z}{\sin 57^\circ} = \frac{21}{\sin 75^\circ}$$

$$z \sin 75^\circ = 21 \sin 57^\circ$$

$$Z = \frac{21 \sin 57^\circ}{\sin 75^\circ} \quad Z = \frac{17.61208193}{\sin 75^\circ}$$

$$\boxed{z = 18.23 \text{ mm (to 2 decimal places)}}$$

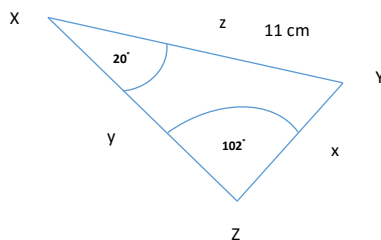
(5 marks)

(b) Triangle XYZ

$X = 20^\circ$

$Z = 102^\circ$

$z = 11 \text{ cm}$



Solving for x (Applying sine rule)

$$\frac{x}{\sin X} = \frac{z}{\sin Z}$$

$$\frac{x}{\sin 20^\circ} = \frac{11}{\sin 102^\circ}$$

$$x \sin 102^\circ = 11 \sin 20^\circ$$

$$x = \frac{11 \sin 20^\circ}{\sin 102^\circ}$$

$$x = \frac{11 \sin 20^\circ}{\sin 102^\circ}$$

$$x = \frac{3.762221577}{0.9781476}$$

$$\mathbf{x = 3.85 \text{ cm (to 2 decimal places)}}$$

Solving for y (Applying sine rule)

$$\frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$Y = 180^\circ - (102^\circ + 20^\circ) = 58^\circ \quad (\text{Internal angles in a triangle sum to } 180^\circ)$$

$$\frac{y}{\sin 58^\circ} = \frac{11}{\sin 102^\circ}$$

$$y \sin 102^\circ = 11 \sin 58^\circ$$

$$y = \frac{11 \sin 58^\circ}{\sin 102^\circ}$$

$$y = \frac{9.328529058}{0.9781476}$$

$$\mathbf{y = 9.54 \text{ cm (to 2 decimal places)}}$$

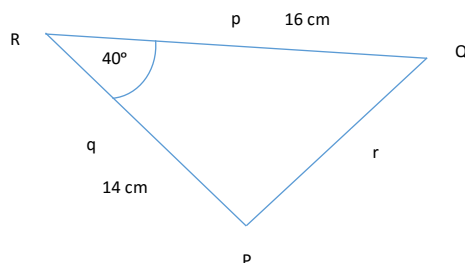
(5 marks)

(c) Triangle PQR

p = 16 cm

q = 14 cm

R = 40°



Solving for r (Applying cosine rule)

$$r^2 = p^2 + q^2 - 2pq \cos R$$

$$r^2 = (16)^2 + (14)^2 - 2(16)(14) \cos R$$

$$r^2 = 256 + 196 - 2(16)(14) \cos R$$

$$r^2 = 452 - 448 \cos 40^\circ$$

$$r^2 = 452 - 343.1879105$$

$$r^2 = 108.8120895$$

$$r = \sqrt{108.8120895}$$

$$\mathbf{r = 10.43 \text{ cm (to 2 decimal places)}}$$

Solving for Q (Applying sine rule)

$$\frac{q}{\sin Q} = \frac{r}{\sin R}$$

$$\frac{14}{\sin Q} = \frac{10.43}{\sin 40^\circ}$$

$$14 \sin 40^\circ = 10.43 \sin Q$$

$$10.43 \sin Q = 14 \sin 40^\circ$$

$$\sin Q = \frac{14 \sin 40^\circ}{10.43}$$

$$\sin Q = 0.86280216$$

$$Q = \sin^{-1}(0.86280216)$$

$$\mathbf{Q = 59.63^\circ \text{ (to 2 decimal places)}}$$

Internal angles in a triangle sum to 180°

$$P = 180^\circ - (59.63^\circ + 40^\circ)$$

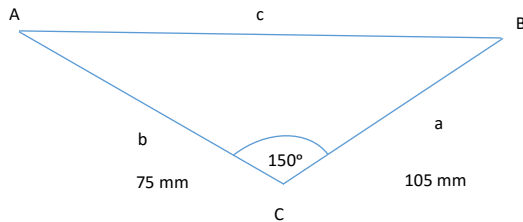
$$P = 180^\circ - 99.63^\circ$$

$$\mathbf{P = 80.37^\circ \text{ (to 2 decimal places)}}$$

(5 marks)

Question 4**Find the area of each triangle below:****(a) Triangle ABC**

$$a = 105 \text{ mm } b = 75 \text{ mm } C = 150^\circ$$

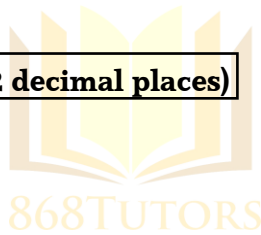


$$\text{Area of triangle} = \frac{1}{2} ab \times \sin C$$

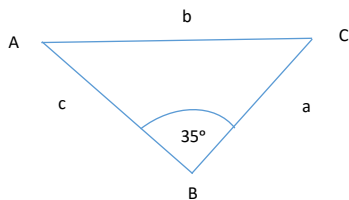
$$\text{Area of triangle} = \frac{1}{2} (105)(75) \times \sin 150^\circ$$

$$\text{Area of triangle} = 3937.5 \times 0.5$$

$$\text{Area of triangle} = 1968.75 \text{ mm}^2 \text{ (to 2 decimal places)}$$

**(5 marks)****(b) Triangle ABC**

$$a = 4.5 \text{ mm } c = 3 \text{ mm } B = 35^\circ$$



$$\text{Area of triangle} = \frac{1}{2} ac \times \sin B$$

$$\text{Area of triangle} = \frac{1}{2} (4.5)(3) \times \sin 35^\circ$$

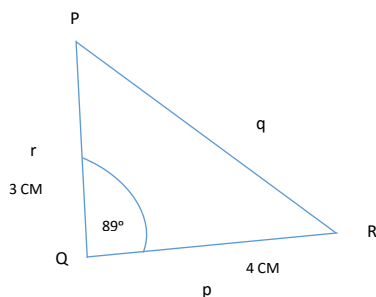
$$\text{Area of triangle} = 6.75 \times \sin 35^\circ$$

$$\text{Area of triangle} = 3.87 \text{ mm}^2 \text{ (to 2 decimal places)}$$

(5marks)

(c) Triangle PQR

$$p = 4 \text{ cm } r = 3 \text{ cm } Q = 89^\circ$$



$$\text{Area of triangle} = \frac{1}{2} pr \times \sin Q$$

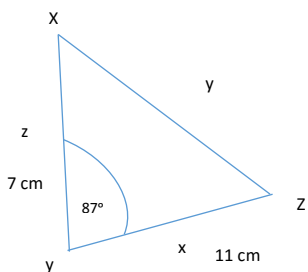
$$\text{Area of triangle} = \frac{1}{2} (4)(3) \times \sin 89^\circ$$

$$\text{Area of triangle} = 6 \times \sin 89^\circ$$

$$\text{Area of triangle} = 6.00 \text{ cm}^2 \text{ (to 2 decimal places)}$$

**(5 marks)****(d) Triangle XYZ**

$$x = 11 \text{ cm } z = 7 \text{ cm } Y = 87^\circ$$



$$\text{Area of triangle} = \frac{1}{2} xz \times \sin Y$$

$$\text{Area of triangle} = \frac{1}{2} (11)(7) \times \sin 87^\circ$$

$$\text{Area of triangle} = 38.5 \times \sin 87^\circ$$

$$\text{Area of triangle} = 38.45 \text{ cm}^2 \text{ (to 2 decimal places)}$$

(5marks)



END OF WORKSHEET



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