

868



TUTORS

Preparation for

High School Mathematics

Functions

Solutions

Math



Instructions and Tips:

- ✓ **You have 60 minutes to complete this worksheet**
- ✓ **This worksheet consists of 6 questions**
- ✓ **Write answers in the spaces provided**
- ✓ **Show all working**



Student Name: _____

Student ID: _____

Date: __ / __ / ____

Total Score:

Highest Score:

Tutor's Comments:

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Question 1

Given that $f(x) = x + 1$ and $g(x) = \frac{2}{x}$

(a) Calculate $f(2)$.

$$f(x) = x + 1$$

$$f(2) = 2 + 1$$

$$f(2) = 3$$

(b) Calculate $g(2)$.

$$g(x) = \frac{2}{x} \quad g(2) = \frac{2}{2}$$

$$g(2) = 1$$

(c) Calculate $g(-1)$.

$$g(x) = \frac{2}{x} \quad g(-1) = \frac{2}{-1}$$

$$g(-1) = -2$$

(d) Write an expression for $gf(x)$.

$$gf(x) = \frac{2}{x+1}$$

(e) Write an expression for $fg(x)$.

$$fg(x) = \frac{2}{x} + \frac{1}{1}$$

$$fg(x) = \frac{2+x}{x}$$



(f) Calculate $fg(2)$.

$$f(x) = x + 1$$

$$g(x) = \frac{2}{x}$$

$$fg(x) = \frac{2+x}{x} \quad fg(2) = \frac{2+2}{2}$$

$$\boxed{fg(2) = 2}$$

(g) Calculate the values of x so that $f(x) = g(x)$.

$$f(x) = x + 1 \quad g(x) = \frac{2}{x}$$

$$x + 1 = \frac{2}{x}$$

$$\frac{x+1}{1} = \frac{2}{x} \quad (\text{By cross-multiplication})$$

$$x(x+1) = 2$$

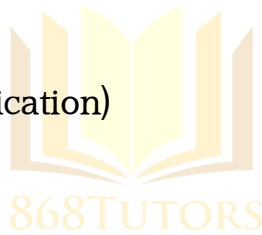
$$x(x+1) = 2$$

$$x^2 + x = 2$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$\boxed{x = -2 \text{ or } x = 1}$$



(10 marks)

Question 2

Given that $f(x) = x + 3$ and $g(x) = \frac{4}{x}$

(a) Calculate $f(-2)$.

$$f(x) = x + 3$$

$$f(-2) = -2 + 3$$

$$f(-2) = 1$$

(b) Calculate $g(2)$.

$$g(x) = \frac{4}{x} \quad g(2) = \frac{4}{2}$$

$$g(2) = 2$$

(c) Calculate $g(-2)$.

$$g(-2) = \frac{4}{-2}$$

$$g(-2) = -2$$



(d) Write an expression for $gf(x)$.

$$gf(x) = \frac{4}{x+3}$$

(e) Write an expression for $fg(x)$.

$$fg(x) = \frac{4}{x} + \frac{3}{1}$$

$$fg(x) = \frac{4+3x}{x}$$

(f) Calculate $fg(1)$.

$$fg(x) = \frac{4 + 3x}{x} \quad fg(1) = \frac{4 + 3(1)}{1}$$

$$\boxed{fg(1) = 7}$$

(g) Calculate the values of x so that $f(x) = g(x)$.

$$f(x) = x + 3 \quad g(x) = \frac{4}{x}$$

$$x+3 = \frac{4}{x}$$

$$\frac{x+3}{1} \Rightarrow \frac{4}{x} \quad (\text{By cross-multiplication})$$

$$x(x+3) = 4$$

$$x^2 + 3x = 4$$

$$x^2 + 3x - 4 = 0$$

$$(x+4)(x-1) = 0$$

$$\boxed{x = -4 \text{ or } x = 1}$$



(10 marks)

Question 3**Given that $f(x) = x^2$ and $g(x) = x - 3$** **(a) Calculate $g(6)$.**

$$g(x) = x - 3$$

$$g(6) = 6 - 3$$

$$\boxed{g(6) = 3}$$

(b) Calculate $g^{-1}(4)$.

$$g(x) = x - 3$$

$$\text{let } y = x - 3$$

$$x - 3 = y \quad (\text{replace } x \text{ with } y \text{ and replace } y \text{ with } x)$$

$$y - 3 = x$$

$$y = x + 3 \quad (\text{make } y \text{ the subject of the formula})$$

$$g^{-1}(x) = x + 3$$

$$g^{-1}(4) = 4 + 3$$

$$\boxed{g^{-1}(4) = 7}$$

(c) Write an expression for $fg(x)$.

$$fg(x) = (x-3)^2$$

$$fg(x) = (x-3)(x-3)$$

$$fg(x) = x^2 - 3x - 3x + 9$$

$$\boxed{fg(x) = x^2 - 6x + 9}$$

(d) Calculate $fg(0)$.

$$fg(x) = x^2 - 6x + 9$$

$$fg(0) = (0)^2 - 6(0) + 9$$

$$\boxed{fg(0) = 9}$$

(6 marks)

Question 4**Given that $f(x) = \frac{x}{2} - 1$ and $g(x) = 3x + 1$** **(a) Calculate $g(3)$.**

$$g(x) = 3x + 1$$

$$g(3) = 3(3) + 1$$

$$\boxed{g(3) = 10}$$

Express in its simplest form**(b) $f^{-1}(x)$**

$$f(x) = \frac{x}{2} - 1$$

$$\text{let } y = \frac{x}{2} - 1$$

$$x = \frac{y}{2} - 1 \quad (\text{replace } x \text{ with } y \text{ and replace } y \text{ with } x)$$

$$\frac{y}{2} - 1 = x \quad (\text{make } y \text{ the subject of the formula})$$

$$\frac{y}{2} = x + 1$$

$$\frac{y}{2} = \frac{x + 1}{1}$$

$$y = 2(x + 1)$$

$$\boxed{f^{-1}(x) = 2(x + 1)}$$

(c) $g^{-1}(x)$

$$g(x) = 3x + 1 \quad \text{let } y = 3x + 1$$

$$x = 3y + 1 \quad (\text{replace } x \text{ with } y \text{ and replace } y \text{ with } x)$$

$$3y + 1 = x \quad 3y = x - 1 \quad y = \frac{x - 1}{3} \quad (\text{make } y \text{ the subject of the formula})$$

$$\boxed{g^{-1}(x) = \frac{x - 1}{3}}$$

(d) $fg(x)$

$$f(x) = \frac{x}{2} - 1 \quad g(x) = 3x + 1$$

$$fg(x) = \frac{3x+1}{2} - 1$$

$$fg(x) = \frac{3x+1}{2} - \frac{2}{2}$$

$$fg(x) = \frac{3x+1-2}{2}$$

$$\boxed{fg(x) = \frac{3x-1}{2}}$$

(e) $(fg)^{-1}(x)$

$$fg(x) = \frac{3x-1}{2}$$

$$\text{let } y = \frac{3x-1}{2}$$

$$x = \frac{3y-1}{2} \quad (\text{replace } x \text{ with } y \text{ and replace } y \text{ with } x)$$

$$\frac{x}{1} = \frac{3y-1}{2} \quad (\text{make } y \text{ the subject of the formula})$$

$$\frac{x}{1} \times \frac{2}{2} = \frac{3y-1}{2} \quad (\text{By cross-multiplication})$$

$$3y-1 = 2x$$

$$3y = 2x + 1$$

$$y = \frac{2x+1}{3}$$

$$\boxed{(fg)^{-1}(x) = \frac{2x+1}{3}}$$

(10 marks)

Question 5**Given that $f(x) = x + 2$ and $g(x) = 3x + 4$** **(a) Calculate $f(4)$.**

$$f(x) = x + 2$$

$$f(4) = 4 + 2$$

$$f(4) = 6$$

Express in its simplest form**(b) $f^{-1}(x)$**

$$f(x) = x + 2$$

$$\text{let } y = x + 2$$

$$x = y + 2 \quad (\text{replace } x \text{ with } y \text{ and replace } y \text{ with } x)$$

$$y = x - 2 \quad (\text{make } y \text{ the subject of the formula})$$

$$f^{-1}(x) = x - 2$$

(c) $g^{-1}(x)$

$$g(x) = 3x + 4$$

$$\text{let } y = 3x + 4$$

$$x = 3y + 4 \quad (\text{replace } x \text{ with } y \text{ and replace } y \text{ with } x)$$

$$3y + 4 = x \quad (\text{make } y \text{ the subject of the formula})$$

$$3y = x - 4$$

$$y = \frac{x - 4}{3}$$

$$g^{-1}(x) = \frac{x - 4}{3}$$

(d) $fg(x)$

$$f(x) = x + 2 \quad g(x) = 3x + 4$$

$$fg(x) = 3x + 4 + 2$$

$$\boxed{fg(x) = 3x + 6}$$

(e) $(fg)^{-1}(x)$

$$fg(x) = 3x + 6$$

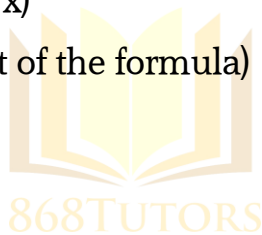
$$\text{let } y = 3x + 6$$

$$x = 3y + 6 \quad (\text{interchange } y \text{ and } x)$$

$$3y = x - 6 \quad (\text{make } y \text{ the subject of the formula})$$

$$y = \frac{x-6}{3}$$

$$\boxed{(fg)^{-1}(x) = \frac{x-6}{3}}$$

**(f) Show that $(fg)^{-1}(x) = g^{-1}f^{-1}(x)$**

Required to show that $(fg)^{-1}(x) = g^{-1}f^{-1}(x)$

$$(fg)^{-1}(x) = \frac{x-6}{3}$$

$$\text{recall from (c) } g^{-1}(x) = \frac{x-4}{3} \quad \text{recall from (b) } f^{-1}(x) = x - 2$$

$$g^{-1}f^{-1}(x) = \frac{x-2-4}{3} \quad g^{-1}f^{-1}(x) = \frac{x-6}{3}$$

Therefore $(fg)^{-1}(x) = g^{-1}f^{-1}(x)$ has been proven.

(10 marks)

Question 6

Given that $g(x) = 2 - x$ and $f(x) = x^3$

(a) Calculate $f(-3)$.

$$f(x) = x^3$$

$$f(-3) = (-3)^3 = -3 \times -3 \times -3$$

$$\boxed{f(-3) = -27}$$

(b) Calculate $fg(3)$.

$$fg(x) = (2 - x)^3$$

$$fg(3) = (2 - 3)^3$$

$$fg(3) = (2 - 3)^3$$

$$fg(3) = (-1)^3$$

$$\boxed{fg(3) = -1}$$



(c) Calculate $gf(3)$.

$$gf(x) = 2 - x^3$$

$$gf(3) = 2 - (3)^3$$

$$gf(3) = 2 - 27$$

$$\boxed{gf(3) = -25}$$

(6 marks)



END OF WORKSHEET



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